Integer cube roots 24 digits

Answer: 8 digits, lowest possibility:
46415889

Tools

- My knowledge of all the two digit cubes, so 01 – 99 by heart
- A bit of algebra, the $(a+b)^3$ formula
- Modulo 33 calculation

Question number

1405 8802 1090 3386 2253 3153

Elaboration

- $132651(51^3) < 140588 < 140608(52^3)$

so the first 2 digits of the answer are 51

Last two digits Q(uestion N(umber)

53 means that the last two digits of the answer are 37, as $37^3 = 50653$

Now answer so far 51 ?? ?? 37

Cubing means….

All the below hundred ending on 1,3,7,9 cubed,

their last two digits (mod 100) have a different answer, also ending on 1,3,7,9 so only one correct answer is possible

The “jump”

- This term, invented by myself indicates the increase of the hundreds in powering according to the formula $(a+b)^3$ gives:

General formula+

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

we now take 137 where 37 = a and 100 = b

Then $3a^2b = 3 \times 37^2 \times 100 = 410700$ of which we take the 0700 as the jump, so for every hundred the hundreds cubed increase with

0700

Then

- We take the last 4 digits of the q.n.(question number) 3153 and subtract the last four digits cube of 37^3 = (5) 0653 and get 2500
- And divide 2500 by 700, or (5)25 ÷ 7 which is 75(mod 100) as 75 x 7 = 525

A.N.(answer number, part)

- We had already 51 and take the 75 we just calculated and now have the last four digits of the answer,
- Now answer so far 51 ?? 7537

The formula (again)

- 140608- 132651 = (rounded) 7950
- 140588– 132651 = (rounded) 7900
- 7900 ÷ 8200 = (rounded) ± 0,99

Modulo 33

- The (n^3) modulo 33 of QN:

  1405 8802 1090 3386 2253 3153

- 18 + 9 + 20 + 1 + 24 + 19 = 91 = 25(33)
- So the answer number (n) is 31 (33) I know this table by heart or calculate quickly
- Answer so far is 51 + ?? + 75 + 37 = 163 = 31(33)

Modulo 33

- 3153 = 18(33) because 31 + 53 = 84 = 18(33)

  In modulo calculation I always work from right to left as a steady method, to avoid problems in the case of odd numbers and by adding the mod 33 of all the four digit groups I have the mod 33 of the complete number.

Final answer

- We “miss” 33(33) , in fact 0(33) and the possibilities are 00,33,66 and 99
- As we before calculated that the difference between 140608 and 140588 has a ratio of ~ 0,99 we take 99 so our final answer is

51 99 75 37

The cubic fives

It is generally known: calculating cubic roots is much easier for odd numbers. Why? Well, there is only one correct answer possible, e.g. 379. If we take three digit even numbers, there are in principle four possibilities, e.g. 28, 278, 528 and 778. Later on I'll try to find a method for this, in this article I'll write about the numbers ending on five. We are forced to have a look on 4 or 5 final digits.
My intention was to do the work out without any paperwork, this appeared to be impossible, for me at least.

Hereunder you'll find the table with the cubic fives up to 1.000, which is leading for my elaboration. BN means basic number.

BN	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**
5	125	105	1157625	205	8615125	305	28372625	405	66430125
15	3375	115	1520875	215	9938375	315	31255875	415	71473375
25	15625	125	1953125	225	11390625	325	34328125	425	76765625
35	42875	135	2460375	235	12977875	335	37595375	435	82312875
45	91125	145	3048625	245	14706125	345	41063625	445	88121125
55	166375	155	3723875	255	16581375	355	44738875	455	94196375
65	274625	165	4492125	265	18609625	365	48627125	465	100544625
75	421875	175	5359375	275	20796875	375	52734375	475	107171875
85	614125	185	6331625	285	23149125	385	57066625	485	114084125
95	857375	195	7414875	295	25672375	395	61629875	495	121287375

BN	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**	**BN**	**Cubic**
505	128787625	605	221445125	705	350402625	805	521660125	905	741217625
515	136590875	615	232608375	715	365525875	815	541343375	915	766060875
525	144703125	625	244140625	725	381078125	825	561515625	925	791453125
535	153130375	635	256047875	735	397065375	835	582182875	935	817400375
545	161878625	645	268336125	745	413493625	845	603351125	945	843908625
555	170953875	655	281011375	755	430368875	855	625026375	955	870983875
565	180362125	665	294079625	765	447697125	865	647214625	965	898632125
575	190109375	675	307546875	775	465484375	875	669921875	975	926859375
585	200201625	685	321419125	785	483736625	885	693154125	985	955671625
595	210644875	695	335702375	796	504358336	895	716917375	995	985074875

We start with 5^3 and add always 100 and study the "jump", the number of hundreds with which the hundreds increase, for the last five digits. So for 5^3, 105^3 and 205^3 and so on we see: 125, 57625, 15125, 72625 etc. Conclusion for the 05 the jump with every hundred is 57500.
For the fifteens we see this in the third power: 03375, 20875, 38375, 55875. There the jump is 17500 per hundred.
Then the 25-s. The last five digits in the third power are: 15625, 53125, 90625, 28125. So here the jump is 37500 per 100.

Studying the row from 05 up to 95 we see that the jumps are: 57500, 17500, 37500, 17500, 57500, 57500, 17500, 37500,, 17500, 57500. And we conclude: for (0)5, 45, 55 and 95 the jumps per 100 are 57500, and also 5+95=100 and 45+55=100.

For 25 and 75 the jumps are 37500, 25+75=100. And for 15, 35, 65 and 85 the jumps per 100 are 17500, and 15+85=100 and 35+65=100.
Another conclusion: as every 7500 × 4 gives another tenthousand, which we see e.g. in 35^3 = 42875 and 435^3 ends on 12875, the tenthousands differ 70.000 which is 4×17500. Working with this we can find the two last digits of the answer.

This we need to know for extracting integer cube roots, in fact we need to know the cubes up to 100 for successfully doing this for numbers up to 24 digits.

As the final answer is calculated by means of modulo 33 calculation, it is primordial to calculate the first four digits of the answer. After that we have the first four digits of the answer, and as we have the last two digits of the answer, we can find the missing digits by means of modulo 33 calculation. So to be confident with modulo 33 calculation is another requirement.

Elaboration of the cube root of 8095 2460 4166 6208 8571 5375.
The beginning is easy: 93^3 = 804357, 94^3 = 830584 so the first digits of the answer are 93.

Now it is getting difficult. For the reliable calculation of the following two digits I use the Newton method which always gives two exact digits, which is enough for our purpose . 809524 – 804357 = 5167 and 830584 – 804357 = 26227; 5167 ÷ 26227 = 0,198, now answer so far 9319.

Now the modulo 33 calculation, in which I firstly calculate the mod 33 of every part of four digits, add the result and then find the modulo 33 of the question number. We go from right to left and find: n^3 =29+24+4+8+18+10 = 93 = 27, so n is 3 mod 33.

Next the last two digits. As the last digits of the n^3 are 5375, we conclude that the origin is 35, as 35^3 = 42875 + a multiple of 57500. We think modulo 10000 and do 15375 – 42875 = 72500, and see that the answer will be 3 modulo 4.

Answer so far 93 19 ?? 35, 147, which is 15 mod 33. The final answer is 3 mod 33, so we have to find 21 mod 33, which means our answer can be either 21, 54 or 87 mod 33. As 87 is the only number which is 3 mod.4 we destine our final answer 93 19 87 35.

Another example: cube root of 1637 3595 6175 5895 2236 1125.
As 157464< 163735<166375 the first two digits of the answer are 54.
Now Newton: 163735- 157464 = 6271 and 166375 – 157464 = 8911; 6271÷ 8911 = 0,703, now the first four digits of our answer are 54 70. The last four digits 1125 bring us immediately to 45, now we have 54 70 ?? 45. Mod 33 of the q.n. = 3+25+21+4+31+20=104, which is 5(33) and this means the mod 33 of the answer will be 14(33) as 14^3 – 2744 > 5(33). 54+70+45=169=4(33), to get 14(33) we need 10(33). The possibilities are 10, 43 and 76.
61125 – 91125 = 70000, the only possibility to get a full 10000 is a number divisible by four, so our final answer is 54 70 76 45, which indeed is 14(33).

For those who are less confident with modulo 33 calculation I make this table:

1	2	3	4	5	6
n^3	n	n^3	n	n^3	n
1	1	12	12	23	23
2	29	13	7	24	18
3	9	14	20	25	31
4	16	15	27	26	5
5	14	16	25	27	3
6	30	17	8	28	19
7	28	18	6	29	17
8	2	19	13	30	24
9	15	20	26	31	4
10	10	21	21	32	32
11	11	22	22	33	0

To work with: calculate the modulo 33 of the question number, 8095 2460 4166 6208 8571 5375, of which the modulo 33 is n^3 =29+24+4+8+18+10 = 93 = 27, so n is 3 mod 33. You can find this in the fifth column, and immediately right from it you see the answer, so in this case n=3

So far with my “cubic fives”, my next project is the integer cube roots of the even numbers, to start with 24 digits. You hear from me!

I wish you a lot of calculation fun!!! Willem Bouman

The cubic evens

It was bound to happen: after my examination of the “cubic fives”, there must be a possibility to extract cubic roots of even numbers, and I found a method. Of course the 000 numbers are excepted! Immediately admitted, I cannot do this without paperwork, for all clarity: no machine was used!

I used these “tools”, the Newton method forfinding the third and fourth digit of the answer, furthermore modulo 8 and 33 calculation.

As modulo 8 calculation is a new issue, we start with that. We work with three digit numbers, so for every answer number there is a difference of 250, e.g. 6, 256, 506 and 756. If the question number ends on 216 with an even thousand, the options are 6 and 506. If the q.n. has an odd thousand, the options are 256 and 756. Generally: if the q.n. divided by 8 gives an odd result, then the answer number will be “only” even, it is 2^1 .

Work out of 12 2303 8915 9810 6025 9528. The answer will have eight digits and the first two digits are 10.
According to Newton we calculate (1223 – 1000) 223 / (1331-1000) 331 = 0,67, answer so far 1067.

Now the last digits: as 9528 has an odd thousand, the last digits will be 62. Answer so far 10 67 ?? 62. The modulo 33 of the q.n. is 24+19+9+5+26+12= 95. 95 is 29 mod 33, therefore the answer number will be 17(33), so 10+67+??+62 = 139. which is 7 mod 33. Next: to come from 7(33) up to 17(33) we have to find a number 10(33).

Now: the jump for 62^3 is $3×62^2$ > (115)32, so 3200 per hundred. And 62^3 = 238328, of which we use 8328. We subtract 9528 – 8328 = 1200 and ask how many times 3200 ends on 1200. Possibilities: 16, 41, 66 and 91. 41 is 8(33) is the most close option we take the 2 mssing(33) for granted and destine our final answer 10 69 41 62 satisfies the two requirements, possibility and mod 33 we destine the final answer 10 69 41 62.

And 2955 7277 0466 8160 8298 6712 2955 7277 0466 8160 8298 6712
As 287496< 295572 < 300763 the first two digits of the answer are 66.

Next: 295572 – 287496 = 8076 and 300763 – 287496 = 13267; and 8076 / 13267 = 0,609 our answer so far 66 61 because of rounding.

Now 6712 . As 6712 is “only” divisible by 8, we conclude the last digits of our answer are 58

Answer so far 66 61 ?? 58 Ξ 185 Ξ 20 (33) . Mod 33 of the q.n. 13+15+9+4+17+18= 76 Ξ 10(33) and the mod 33 of the answer therefore is 10(33) . So for we 20 up to 10(33) we“miss” 23(33).
The jump: (98)6712-(19)5112 Ξ 1600(mod 10.000). Jump 3 $×58^2×$100 Ξ (100) 9200 and now comes the question: how many times 9200 fit in 1600. Possibilities: 23, 48, 73, 98. As only 213 fits we take this in pour final answer 66 61 23 58.

For them who are less confident with modulo 33 calculation I make a table. In the odd columns you see the mod.33 of n^3, in the even columns you see the basic numbers. E.g. in an odd column (5) you see 27, which is 3^3, and in the even column (6) you see the basic number 3.

1	2	3	4	5	6
N^3	N	N^3	N	N^3	N
1	1	12	12	23	23
2	29	13	7	24	18
3	9	14	20	25	31
4	16	15	27	26	5
5	14	16	25	27	3
6	30	17	8	28	19
7	28	18	6	29	17
8	2	19	13	30	24
9	15	20	26	31	4
10	10	21	21	32	32
11	11	22	22	33	33

It was and remains a very interesting and instructive project for me, I hope for you too!!!

Willem Bouman